

1 Extrapolating atm. μ Flux

Assumption:

Number of atm. muons at a given cut level follows a power law with 2 parameters (Normalization Φ_{tot} , Index α). The underlying pdf (normalized to one) is

$$p(E) = \frac{\alpha - 1}{E_{min}} \left(\frac{E}{E_{min}} \right)^{-\alpha} \quad (1)$$

with E_{min} being the energy at which power law behavior starts ($E_{min} = 10$ TeV in our case). Thus the expected number of muons μ_i in the i -th bin $[E_i, E_{i+1}]$ is

$$\mu_i = \Phi_{tot} \int_{E_i}^{E_{i+1}} dE p(E) = \Phi_{tot} \left(\left[\frac{E_i}{E_{min}} \right]^{1-\alpha} - \left[\frac{E_{i+1}}{E_{min}} \right]^{1-\alpha} \right) \quad (2)$$

We can now build a poisson-likelihood for this hypothesis (2) given an observation \vec{n} .

$$L(\alpha, \Phi_{tot} | \vec{n}) = \prod_{i=1}^N P_i(n_i | \alpha, \Phi_{tot}) \quad (3)$$

$$P_i(n_i | \alpha, \Phi_{tot}) = \frac{\mu_i^{n_i}}{n_i!} e^{-\mu_i} \quad (4)$$

Here N denotes the last bin (highest energy) that has a non-zero observation, since a zero observation may just be due to lack of Corsika statistics at high energies.

We find the best matching hypothesis (2) by minimizing $-\ln L$ w.r.t $\{\alpha, \Phi_{tot}\}$ using Minuit2 (Migrad).

$$-\ln L = C_0 + \sum_{i=1}^N (\mu_i - n_i \cdot \ln \mu_i) \quad (5)$$

$$C_0 = \sum_{i=1}^N \ln(n_i!) \quad (6)$$

Since the minimum of $-\ln L$ and hence the best matching hypothesis do not depend on C_0 (constant for a given observation) we neglect it during minimization.