

1 Improvement in Fast Particle Track Reconstruction
2 with Robust Statistics

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116 **Abstract**

117 The IceCube project has transformed one cubic kilometer of deep natural
118 Antarctic ice into a Cherenkov detector. Muon neutrinos are detected and their
119 direction inferred by mapping the light produced by the secondary muon track
120 inside the volume instrumented with photomultipliers. Reconstructing of the
121 muon track from the observed light is challenging due to noise, light scattering
122 in the ice medium, and the possibility of simultaneously having multiple muons
123 inside the detector resulting from the large flux of cosmic ray muons.

124 This manuscript describes work on two problems: (1) the *track reconstruction*
125 *problem*, in which, given a set of observations, the goal is to recover the
126 track of a muon, and (2) the *coincident event problem*, which is to determine
127 how many muons are active in the detector during a time window. Rather than
128 solving these problems by developing more complex physical models that are
129 applied at later stages of the analysis, our approach is to augment the detector's
130 early reconstruction with data filters and robust statistical techniques. These
131 can be implemented at the level of on-line reconstruction and therefore improve
132 all subsequent reconstructions. Using the metric of median angular resolution, a

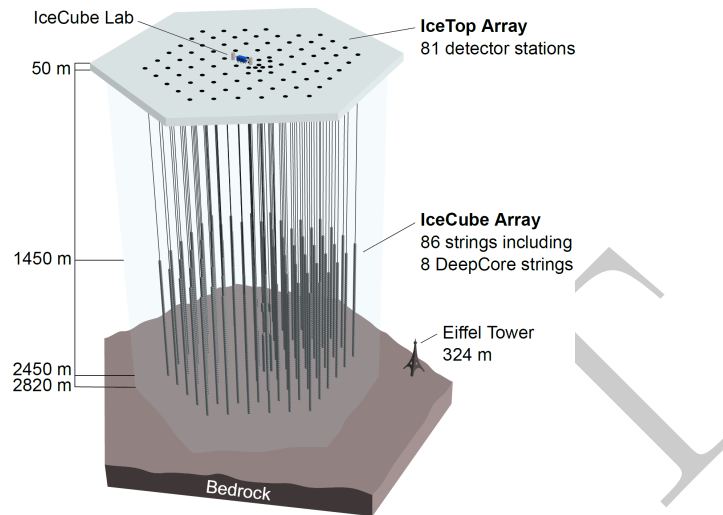


Figure 1: The IceCube neutrino detector in the Antarctic ice. A picture of the Eiffel Tower is shown for scale.

133 standard metric for track reconstruction, we improve the accuracy in the initial
 134 reconstruction direction by 13%. We also present improvements in measuring
 135 the number of muons in coincident events: we can accurately determine the
 136 number of muons 98% of the time, which is an improvement of 86% over the
 137 software previously used in IceCube.

138 *Keywords:* IceCube, Track reconstruction, Neutrino telescope, Neutrino
 139 astrophysics, Robust Statistics

140 **1. Introduction**

141 The IceCube neutrino detector searches for neutrinos that are generated by
 142 the universe’s most violent astrophysical events: exploding stars, gamma ray
 143 bursts, and cataclysmic phenomena involving black holes and neutron stars [1].
 144 The detector, roughly one cubic kilometer in size, is located near the geographic
 145 South Pole and is buried to a depth of about 2.5 km in the Antarctic ice [2].
 146 The detector is illustrated in Figure 1 and a more complete description is given
 147 in Section 2.

148 When a neutrino enters the telescope, it occasionally interacts in the ice and
 149 generates a muon. The neutrino direction can be inferred from a reconstruction
 150 of the muon track. Muons are also generated by cosmic rays interacting
 151 in the atmosphere, and separation of the background of cosmic ray muons and
 152 neutrino-induced muons is a necessary step for neutrino analysis. This separation
 153 is challenging, as the number of observed cosmic ray muons exceeds the

154 number of observed neutrino muons by over five orders of magnitude [3].

155 The primary mechanism for separating the cosmic ray muons from the neu-
156 trino muons is reconstructing the muon track and determining whether the
157 muon was traveling downwards into the Earth or upwards out of the Earth.
158 Since neutrinos can penetrate through the Earth but cosmic ray muons cannot,
159 it follows that a muon traveling out of the Earth must have been generated by a
160 neutrino. Thus, by selecting only the muons that are reconstructed as up-going,
161 the cosmic ray muons can, in principle, be removed from the data. Since the
162 number of cosmic ray muons overwhelms the number of neutrino muons, high
163 accuracy is critical for preventing erroneous reconstruction of cosmic ray muons
164 as neutrino-induced.

165 Here, we examine two problems that arise in the separation of cosmic ray
166 muons from neutrino muons in the IceCube detector:

- 167 1. *Reconstruction*, in which the track of a muon is reconstructed from the
168 observed light at different positions and times in the detector.
- 169 2. *Coincident Event Detection*, in which we detect the number of muons
170 inside the detector, and assign observed photons to a muon.

171 Sophisticated reconstruction techniques have been developed that computa-
172 tionally model in detail the muon's Cherenkov cone as well as the scattering
173 and absorption of photons through layers of Antarctic ice with varying optical
174 properties [3–5]. Rather than further refining these techniques, the current work
175 focusses on improving the statistical techniques and optimizing data filtering in
176 the early online track reconstruction performed on the data in real time at the
177 South Pole. Besides benefiting directly any analysis that uses the online recon-
178 struction such as the search for cosmogenic neutrinos, any later analysis will
179 benefit from improvements made at the early stages of the data collection.

180 1.1. *Related Work*

181 Track reconstruction and coincident event detection challenges are ubiqui-
182 tous in particle physics [6–8], both in particle accelerators and cosmic particle
183 detectors. While the work described in this manuscript builds on the previous
184 technique developed for the IceCube detector [3], these techniques are general
185 purpose, and potentially have applications in detectors beyond IceCube.

186 1.2. *Outline*

187 We begin by describing the IceCube detector and track reconstruction chal-
188 lenges in Section 2. In Section 3, we describe the reconstruction pipeline in-
189 cluding the prior IceCube software, then we present improvements to the online
190 tracking algorithm and discuss the results. Section 4 describes improvements
191 on coincident event detection, and follows a parallel structure to Section 3. We
192 conclude in Section 5.

193 2. IceCube Detector and Track Reconstruction Challenges

194 The IceCube detector is composed of 5,160 optical detectors, each containing
195 a photomultiplier tube (PMT) and onboard digitizer [9]. The PMTs are spread
196 over 86 vertical strings arranged in a hexagonal shape, with a total instrumented
197 volume of approximately one cubic kilometer. The PMTs on a given string
198 are separated vertically by 17 m, and the string-to-string separation is roughly
199 125 m.

200 At an abstract level, IceCube operates by detecting muons as they travel
201 through the instrumented volume of ice. As the muon travels through the
202 detector, it radiates light [4], which is observed by the PMTs and quantized into
203 discrete *hits* [10]. IceCube uses several trigger criteria. The most commonly used
204 trigger selects time intervals where eight PMTs (with local coincidences) fired
205 within 5 microseconds. When a trigger occurs, all data within a 10 microsecond
206 trigger window is saved, becoming an *event*. If the number of hits in an event
207 is sufficiently large, the muon track reconstruction algorithm is triggered.

208 There are several challenges for the reconstruction algorithms used in the
209 detector. Varying optical properties of the ice affect reconstruction accuracy,
210 the data may contain outlier hits due to uncorrelated noise, and there are finite
211 computational resources available to tracking code run on-site.

212 *Modeling Difficulties.* The details of the ice's optical properties are nontrivial to
213 model. Light propagating through the ice is affected by scattering and absorp-
214 tion. These effects cannot be analytically calculated and the optical properties
215 of the ice vary with depth [5].

216 *Noise.* The noise inherent in the data is another challenge. Noise hits can
217 arise either from the thermal background of the photocathode, or from photons
218 generated by radioactive decay inside the PMT [9].

219 *Computational Constraints.* The reconstruction algorithms are also limited in
220 complexity by the computing resources available at the South Pole. The track
221 reconstruction algorithm has to process about 3,000 muons per second, algo-
222 rithms with excessive computational demands are discouraged.

223 3. Reconstruction Improvement

224 As shown in the following, augmenting the reconstruction algorithm with
225 some basic filters and classical data analysis techniques results in significant
226 improvement in the reconstruction algorithm's accuracy.

227 3.1. Prior IceCube Software

228 The muon track reconstruction process (outlined in Figure 2) starts when the
229 number of detected hits exceeds a preset threshold and initiates data collection.
230 After the initial data are collected, the event then passes through a series of
231 basic filters to remove obvious outliers [11].

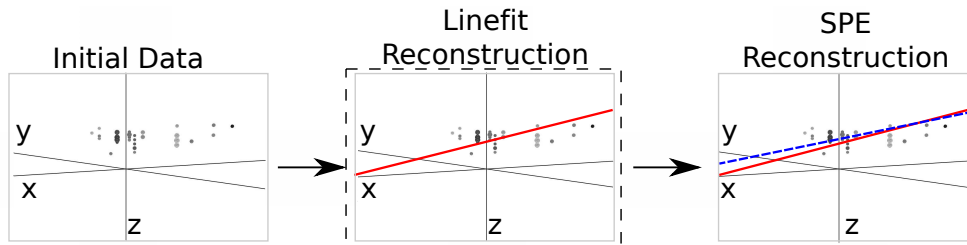


Figure 2: The reconstruction pipeline used to process data in the IceCube detector. After initial data are collected, it is then processed by some basic noise filters, which remove clear outliers. This cleaned data are processed by a basic reconstruction algorithm (solid line), which is used as the seed for the more sophisticated reconstruction algorithm (dashed line). The sophisticated reconstruction is then evaluated as a potential neutrino. The work presented in this manuscript makes changes to the basic reconstruction step (indicated by the dashed box).

232 This is followed by a basic reconstruction algorithm, *linefit* [12], that replaces
 233 the Cherenkov cone by a plane wave and finds the track that minimizes the sum
 234 of the squares of the distances between the track and the hits. More formally,
 235 assume there are N hits; denote the position and time of the i th hit as \vec{x}_i and t_i ,
 236 respectively. Let the reconstructed muon track have a velocity of \vec{v} , and let the
 237 reconstructed track pass through point \vec{x}_0 at time t_0 . Then linefit reconstruction
 238 solves the *least-squares* optimization problem

$$\min_{t_0, \vec{x}_0, \vec{v}} \sum_{i=1}^N \rho_i(t_0, \vec{x}_0, \vec{v})^2, \quad (1)$$

239 where

$$\rho_i(t_0, \vec{x}_0, \vec{v}) = \|\vec{v}(t_i - t_0) + \vec{x}_0 - \vec{x}_i\|_2. \quad (2)$$

240 Linefit is an approximation primarily used to generate an initial track or *seed*
 241 for a more sophisticated reconstruction.

242 The reconstruction algorithm for the sophisticated reconstruction is *Single-*
 243 *Photo-Electron-Fit (SPE fit)* [3]. SPE fit uses the least-squares reconstruction,
 244 the event data, and a parameterized probability distribution function of scatter-
 245 ing in ice [3] to reconstruct the muon track. The SPE fit is the primary
 246 reconstruction algorithm used in the initial data selection and filtering run at
 247 the detector site, and the fit serves as a seed track to the more complex recon-
 248 structions used in off-site data analyses.

249 3.2. Algorithm Improvement

250 SPE fit is dependent on the seed. Given a seed that is inaccurate by 6° or
 251 more, SPE fit typically cannot recover, and produces a reconstruction with the
 252 same level of inaccuracy as the seed track. In addition, the likelihood space for
 253 SPE fit can contain multiple local maxima, so improving the accuracy of a seed
 254 that is already near the true solution improves the accuracy of SPE fit. Thus,
 255 the work here focuses on improving the quality of the seed.

256 As indicated in Equation 1, a least-squares fit models the muon as a single
 257 point moving in a straight line, and hits are penalized quadratically in their
 258 distance from this line. Thus there is an implicit assumption in this model:
 259 that all the hits will be near the muon. This assumption has two pitfalls:

- 260 1. It ignores the scattering effects of the ice medium. Some of the photons can
 261 scatter for over a microsecond, which means that when they are recorded
 262 by a PMT, the muon will be over 300 m away.
- 263 2. While the noise reduction steps remove most of the outlier noise, the noise
 264 hits that survive can be far from the muon. Since these outliers are given
 265 quadratic weight, they exert a huge influence over the model.

266 The first pitfall occurs because the model is incomplete and does not accu-
 267 rately model the data, and the second demonstrates that the model is not robust
 268 to noise. The solution to this is twofold: improve the model and increase the
 269 noise robustness by replacing least squares with robust statistical techniques.

270 3.2.1. Improving the Model

271 The least-squares model does not model the scattering, and thus hits gen-
 272 erated by photons that scattered for a significant length of time are not useful
 273 predictors of the muon’s position. We found that a basic filter could identify
 274 these scattered hits, and improve accuracy by of almost a factor of two by
 275 removing them from the dataset.

276 More formally, for each hit h_i , the algorithm looks at all neighboring hits
 277 within a neighborhood of r , and if there exists a neighboring hit h_j with a time
 278 stamp that is t earlier than h_i , then h_i is considered a scattered hit, and is
 279 not used in the basic reconstruction algorithm. Optimal values of r and t were
 280 found to be 156 m and 778 ns by tuning them on simulated muon data with an
 281 E^{-2} power law spectrum.

282 3.2.2. Adding Robustness to Noise

283 As described in equation 1, the least squares model gives outliers quadratic
 284 weight, whereas we would prefer that outliers had zero weight. There are robust
 285 models in classical statistics designed to marginalize outliers. We determined
 286 that replacing the least-squares model with a Huber fit [13] improves the recon-
 287 struction accuracy.

288 More formally, we replace Equation 1 with the optimization problem:

$$\min_{t_0, \vec{x}_0, \vec{v}} \sum_{i=1}^N \phi(\rho_i(t_0, \vec{x}_0, \vec{v})), \quad (3)$$

289 where the Huber penalty function $\phi(\rho)$ is defined as

$$\phi(\rho) \equiv \begin{cases} \rho^2 & \text{if } \rho < \mu \\ \mu(2\rho - \mu) & \text{if } \rho \geq \mu \end{cases} . \quad (4)$$

Table 1: Median angular resolution (degrees) for reconstruction improvements. The first line is the accuracy of the prior least-squares model, and the subsequent lines are the accuracy measurements from cumulatively adding improvements into the basic reconstruction algorithm.

Algorithm	θ_{med}
Linefit Reconstruction (Least-Squares)	9.917
With Addition of Logical Filter	5.205
With Addition of Huber Regression	4.672
With Addition of Outlier Removal	4.211

290 Here, $\rho_i(t_0, \vec{x}, \vec{v})$ is defined in Equation 2 and μ is a constant calibrated to the
 291 data (on simulated muon events with an E^{-2} power law spectrum, the optimal
 292 value of μ is 153 m).

293 The Huber penalty function has two regimes. In the near-hit regime ($\rho < \mu$),
 294 hits are assumed to be strongly correlated with the muon’s track, and the Huber
 295 penalty function behaves like least squares, giving these hits quadratic weight.
 296 In the far-hit regime ($\rho \geq \mu$), hits are given linear weights as they are more
 297 likely to be noise.

298 In addition to its attractive robustness properties, the Huber fit’s weight
 299 assignment also has the added benefit that it inherently labels points as outliers
 300 (those with $\rho \geq \mu$). Thus, once the Huber fit is computed, we can go one step
 301 farther and simply remove the labeled outliers from the dataset. A better fit is
 302 then obtained by computing the least-squares fit on the data with the outliers
 303 removed. The entire algorithm has a mean runtime that is approximately six
 304 times longer than Linefit’s mean runtime.

305 3.3. Results

306 The goal is to improve the accuracy of the reconstruction in order to better
 307 separate neutrinos from cosmic rays. Thus we present three measurements: (1)
 308 the accuracy change between linefit and the new algorithm, (2) the accuracy
 309 change when SPE is seeded with the new algorithm instead of linefit, and (3)
 310 the improvement in separation between neutrinos and cosmic rays.

311 To measure the improvement generated by the changes, we use the metric of
 312 *median angular resolution* θ_{med} , which is a standard metric within the collab-
 313 oration. The angular resolution of a reconstruction is the arc-distance between
 314 the reconstruction and the true track. The dataset is drawn from simulated
 315 neutrino data designed to be similar to that observed by the detector.

316 We can improve the median angular resolution of the basic reconstruction
 317 by 57.6%, as shown in Table 1. Seeding SPE with the improved basic recon-
 318 struction generates an improvement in the angular resolution of 12.9%. These
 319 improvements in the reconstruction algorithm result in 10% fewer atmospheric
 320 muons erroneously reconstructed as up-going, and 1% more muons correctly
 321 reconstructed as up-going.

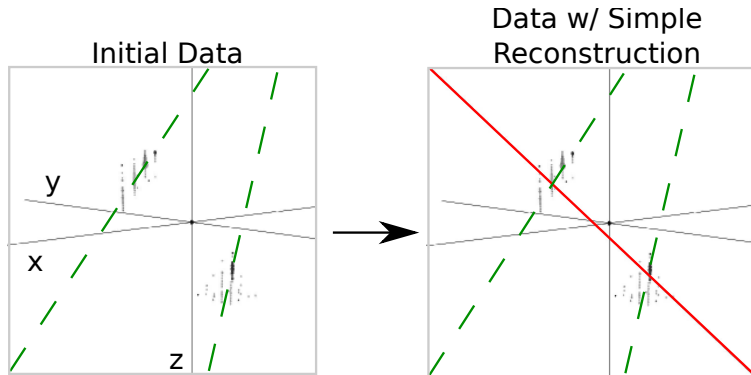


Figure 3: In this example, an event that is clearly composed of two muons (actual tracks shown as dashed lines) is treated as a single muon, and thus the reconstruction (solid line) is inaccurate.

322 4. Coincident Event Improvements

323 In the second study, we look at the problem of determining when more than
 324 one muon has entered the detector. In the most common case, a single muon
 325 will pass through the detector and generate an event before exiting. These events
 326 are processed by the pipeline described in Figure 2. However, for roughly 9%
 327 of the events collected by the data collection algorithm, more than one muon
 328 will be passing through the detector simultaneously, an occurrence known as a
 329 *coincident event*.

330 One of the primary sources of background noise in IceCube analyses is coinci-
 331 dent background muons that have been erroneously reconstructed as neutrinos.
 332 To see why this occurs, consider the coincident event shown in Figure 3. There
 333 are two clear groups of hits; however, the reconstruction algorithm treats them
 334 as a single group, resulting in a erroneous reconstruction. In the ideal case, the
 335 reconstruction algorithm would identify coincident events and split them, as in
 336 Figure 4.

337 The challenge in this example is determining the number of muons in an
 338 event. Our studies show that a simple spatial clustering algorithm can solve
 339 this classification problem with less than 2% error.

340 4.1. Prior IceCube Software

341 Coincident events have been a concern in the IceCube analysis [14] for years,
 342 and some software has been developed to handle coincident events. As a baseline
 343 of comparison, we use the *TTrigger* software, which is described in [15].

344 4.2. Algorithm Improvement

345 Here we present a proximal clustering algorithm. The intuition in proximal
 346 clustering is that points local in space and time are probably from the same

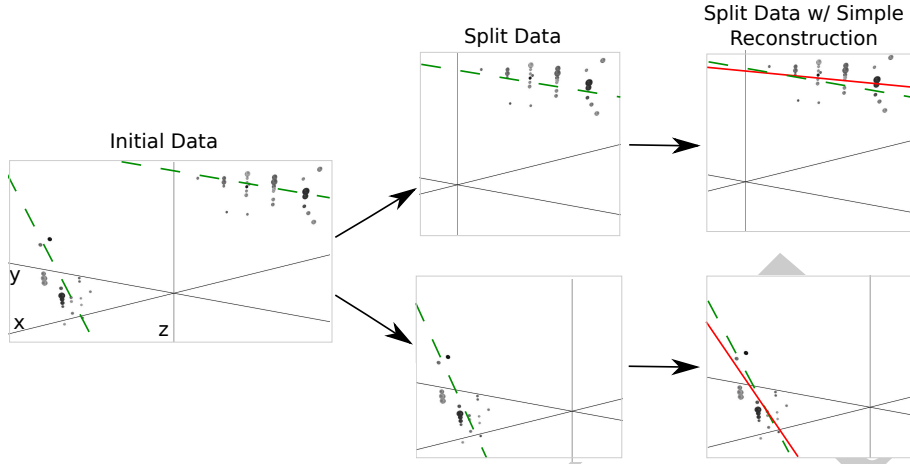


Figure 4: Ideally, the detector would split coincident events before computing the reconstruction. Splitting the event results in more accurate reconstructions (reconstructions shown as solid lines, true muon tracks shown as dashed lines). Note the difference in the reconstructions compared with Figure 3.

347 muon. The proximal clustering algorithm iterates through each pair of hits
 348 (i, j) and builds an adjacency matrix \mathbf{A} as

$$\mathbf{A}_{ij} = \begin{cases} 1 & \text{if } \|\Delta x^2 + \Delta y^2 + \Delta z^2 + (c\Delta t)^2\|_2 \leq \alpha, \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

349 where $\Delta x, \Delta y, \Delta z$ and Δt are the space and time differences between the pair
 350 of hits, and α is tuned to the data (in this application, the optimal value of
 351 α is 450 m). The clustering can be recovered by extracting the connected
 352 components of the graph defined by \mathbf{A} . A connected component of a graph is a
 353 subgraph such that there exist a path between any two vertices of this subgraph.

354 4.2.1. Improving the Model

355 When implemented naively, proximal clustering succeeded for the majority
 356 of the events, but failed if there was a gap in the muon track, which can occur
 357 when the muon travels through dusty ice layers with short scattering length. If
 358 there is a significantly large gap, the algorithm erroneously separates the hits
 359 into two clusters.

360 To compensate, an additional heuristic is added, *track connecting*. After the
 361 data segmentation is finished, track connecting determines if separate clusters
 362 should be combined. It computes the mean position and time of each cluster,
 363 and connects a hypothetical muon track T between each pair of subspaces.

364 It checks if the speed s of the hypothetical track is within 25% of the speed
 365 of light c , and it checks that the mean distance between hits and T in both
 366 clusters is less than 60 m. If T passes both checks, the clusters are combined.

367 *4.2.2. Adding Robustness to Noise*

368 Proximal clustering is susceptible to noise. Noise hits close to two disjoint
369 tracks will be considered adjacent to both tracks, and thus can connect the two
370 tracks in the adjacency matrix.

371 One heuristic that worked well at mitigating this problem was to not use
372 all the hits in building the adjacency matrix. During data collection, some hits
373 are flagged as having a *local coincidence condition*, which indicates that both
374 they and a neighboring PMT reported a hit. These hits have a high probability
375 of not being noise hits, and thus exclusively using them to build the adjacency
376 matrix mitigates the problem of erroneously connecting two tracks.

377 After the proximal clustering algorithm has extracted the tracks from the
378 adjacency matrix, the hits not used in the construction of the adjacency matrix
379 are simply assigned to the closest reconstructed track.

380 *4.3. Results*

381 There were two competing goals for coincident event detection algorithms:
382 the algorithm should be conservative enough that events containing single tracks
383 are not erroneously split, and aggressive enough that a useful fraction of coin-
384 cident events are split correctly. Our algorithm is tuned to keep almost all
385 of the single events correctly unsplit, while still correctly splitting 80% of the
386 coincident events.

387 *4.3.1. Measurements*

388 We modified the reconstruction pipeline shown in Figure 2, in between the
389 noise cleaning and the basic reconstruction, by adding a step for coincident event
390 detection, as shown in Figure 4. This step takes cleaned data and attempts to
391 classify the event as a single-track or multiple-track event.

392 We ran each algorithm on two datasets of simulated data. One dataset
393 comprised single-muon events, and the other dataset comprised multiple-muon
394 events. In each dataset, we measured the classification error E , which is the
395 fraction of events that were misclassified. To get a global measurement, we
396 compute the *total error* E_{tot} , defined as

$$E_{tot} = w_{\text{Single}}E_{\text{Single}} + w_{\text{Multiple}}E_{\text{Multiple}}. \quad (6)$$

397 For computing E_{tot} , we use $w_{\text{Single}} = 0.917$ and $w_{\text{Multiple}} = 0.083$, which is
398 the frequency in which single-muon and multiple-muon events appear in data
399 simulating the distribution of events that trigger the reconstruction algorithm.

400 We present the results for the coincident event problem by measuring how
401 well each algorithm performs at determining the number of subspaces in an
402 event.

403 There are two natural comparisons for the work: the prior software TTrigger,
404 as well as the trivial algorithm, which always classifies each event as a single-
405 track event. Clearly, the latter will always get the single-track events correct,
406 and always get the multiple-track events wrong. We provide a comparison of
407 these techniques in Table 2. As shown, the new algorithm classifies the number
408 of muons in the detector 86% better than TTrigger.

Table 2: Error Rates for Classification Algorithms

Algorithm	$E_{\text{Single}} \%$	$E_{\text{Multiple}} \%$	$E_{\text{tot}} \%$
Trivial	0.0	100.0	8.3
TTrigger	11.5	31.8	13.2
Proximal clustering	0.2	18.9	1.8

409 5. Conclusions

410 We found that significant improvements can be achieved in the IceCube’s on-
 411 line track reconstruction by employing some classical data analysis algorithms.
 412 Optimizing data filtering and refining the least-square model have led to signif-
 413 icant improvements in the accuracy of the reconstruction direction. The new
 414 reconstruction software is fast enough to run on-site, and is now included in all
 415 IceCube analyses.

416 We also looked at the problem of determining the number of muons in the
 417 detector. We found that proximal clustering with some basic heuristics could
 418 correctly determine whether an event contained a single muon or multiple muons
 419 with less than 2% error, yielding an 86% improvement over the prior software.

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